Chapter 2

Settling characteristics of suspensions

2.1 INTRODUCTION

Waters and wastewaters contain a wide variety of particulate matter, the component particles varying in size, shape and specific gravity. The concentration of particles may also vary widely. Particle concentration may be expressed in volumetric or gravimetric terms.

As discussed in section 1.3.4, the specific gravity of particles is determined by the density of the particle solid matter and the amount of entrained water. Organic particles, in particular, typically entrain large amounts of water (see section 1.1). In water processes engineering the concentration of particles is mostly expressed in w/v terms, as mg l^{-1}. However, in particle settling analysis and in the design of particle separation processes, the volumetric concentration (ml l^{-1}) is also of significance. Where the volumetric concentration is high, the rate of individual particle settling is hindered by the upward movement of water displaced by the downward movement of suspended particles.

In the following sections, three categories of particle settling behaviour are examined: (a) discrete particle settling, where the mutual influence of adjacent particles is negligible; (b) hindered particle settling, which applies to suspensions where the volumetric suspension concentration exceeds a certain threshold value; and (c) so-called ‘zone settling’ behaviour, which is exhibited by flocculent suspensions that have a sufficiently high volumetric concentration to settle with a well-defined interface between the suspension and the supernatant water.

2.2 SETTLING OF DISCRETE PARTICLES

The forces acting on a discrete particle settling in a water mass are its weight force, F_w, a buoyancy force, F_b, and a fluid drag force, F_d. hence, the rate of change of momentum of a discrete particle of mass m, settling in still water, is defined by these forces as follows:

$$m\frac{dv}{dt} = F_w - F_b - F_d \tag{2.1}$$

where

the weight force:	$F_w = \rho_s gV$, ρ_s being the particle density and V the particle volume
the buoyancy force:	$F_b = \rho gV$, ρ being the water density
the drag force:	$F_d = C_d A\rho v^2/2$, C_d being the drag coefficient, A is the projected area of the particle in the direction of motion, and $\rho v^2/2$ is the dynamic or stagnation point pressure.

On inserting these values for F_w, F_b and F_d, equation (2.1) becomes

$$m\frac{dv}{dt} = gV(\rho_s - \rho) - C_d A\rho v^2/2 \tag{2.2}$$

The motion of a particle in a centrifuge is subject (neglecting its weight force) to a corresponding set of forces and may be represented by the following equation:

$$m\frac{dv}{dt} = aV(\rho_s - \rho) - C_d A\rho v^2 / 2 \tag{2.3}$$

where a is the radial acceleration, $a = r\omega^2$, r denoting radius and ω the angular velocity.

As indicated by equation (2.2), it is clear that in gravitational settling, the settling velocity increase until the drag force becomes equal to the submerged weight force. From inspection of equation (2.3) it is clear that, in centrifuging, particle velocity does not converge rapidly to a terminal value because of the fact that a, the radial acceleration, increase with r, the distance of the particle from the centre of rotation.

In gravitational settling (sedimentation), the particle reaches its terminal settling velocity as the accelerating force is reduced to zero:

$$m\frac{dv}{dt} = 0 = gV(\rho_s - \rho) - C_d A\rho v^2 / 2$$

For spherical particles, $A = \pi d^2/4$ and $V = \pi d^3/6$ and hence the value of the terminal settling velocity v_t is:

$$v_t = \left[\frac{1.33gd(\rho_s - \rho)}{C_d \rho}\right]^{0.5} \tag{2.4}$$

The drag coefficient, C_d, depends on the state of flow around the particle, which may be laminar, transitional or turbulent as may be categorized by the particle Reynolds number R_e, which is defined as follows:

$$R_e = \rho d v_t / \mu \tag{2.5}$$

The appropriate R_e value ranges for the flow types are:

laminar:	if $R_e < 1$
transitional:	if $1 < R_e < 2000$
turbulent	if $R_e > 2000$

Under laminar conditions $C_d = 24/Re$, giving the Stokes (1845) expression for terminal settling velocity:

$$v_t = \frac{gd^2}{18\mu}(\rho_s - \rho) \tag{2.6}$$

or

$$v_t = \frac{gd^2}{18\nu}(S_g - 1) \tag{2.7}$$

where $S_g = \rho_s/\rho$ is the particle specific gravity and $\nu = \mu/\rho$ is the liquid kinematic viscosity.

Under turbulent conditions C_d is independent of R_e and may be taken to have a value of 0.4 for spherical particles, giving the following expression for turbulent settling velocity:

$$v_t = \left[3.33gd(S_g - 1)\right]^{0.5} \tag{2.8}$$

For the transitional region, fair and Geyer (1968) suggested the following empirical expression:

$$C_d = 24/R_e + 3/(R_e)^{0.5} + 0.34 \qquad (2.9)$$

For all flow conditions other than laminar the drag coefficient is also a function of the shape of the particle and must be determined experimentally. Non-spherical particles will settle more slowly than spherical particles of the same volume and density.

The relationships in the foregoing expressions for the drag coefficient C_d and the terminal settling velocity v_t are summarized graphically in Figs 2.1 and 2.2.

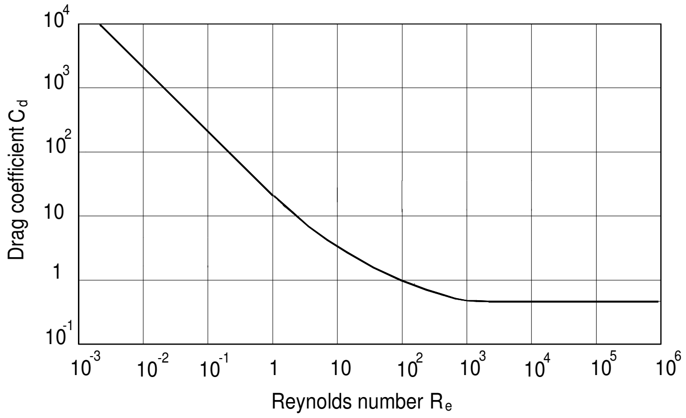

Fig 2.1 **Drag coefficient for settling of spherical particles**

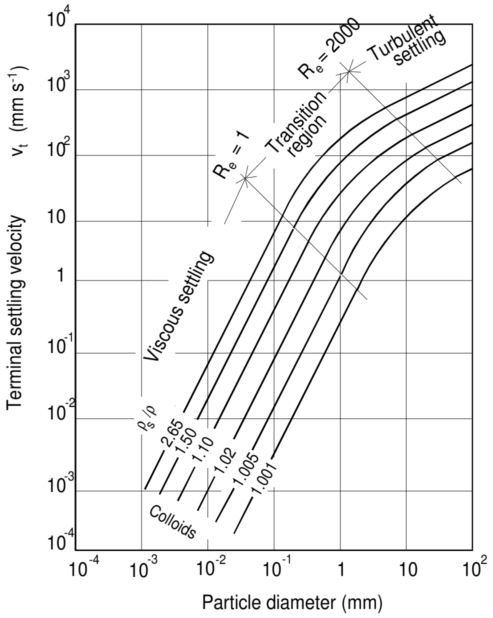

Fig 2.2 **Quiescent settling of spherical particles (at water temperature of 10 °C)**

2.3 HINDERED SETTLING OF DISCRETE PARTICLES

In a suspension in which particles are widely separated, the upward flow of displaced water resulting from the downward movement of an individual particle will not affect the rate of settling of neighbouring particles. However, as the volumetric concentration of particles increases, the particles will begin to restrict the area through which the displaced liquid moves upward, thus hindering downward movement. Settling under such conditions is designated as hindered settling.

The hindered settling velocity of a particle, v_h, may be related to its unhindered settling velocity, v_t, by a relationship of the form

$$v_h = v_t\phi(c_v) \tag{2.10}$$

where ϕ (c_v) denotes a function of the volumetric suspension concentration c_v. For organic flocculent suspensions of the type encountered in water and wastewater treatment, the value of c_v is very much influenced by the amount of entrained water in the particles that comprise these suspensions. The volumetric concentration is related to the gravimetric concentration c as follows:

$$c_v = \frac{c}{\rho_s} + \left(\frac{100-P}{P}\right)\left(\frac{c}{\rho}\right) \tag{2.11}$$

where P is the percentage of dry matter in the particle. Take, for example, an organic particulate suspension where the solid fraction has a specific gravity of 1.4 and the gravimetric concentration is 1000 mg l^{-1} (1 kg m^{-3}). The volumetric concentration is calculated from equation (2.11) to vary with particle water content as follows:

% dry matter	10	1	0.1
Volumetric concentration, c_v	0.00971	0.09971	0.99971

The following analytical quantification if $\phi(c_v)$ (Bond, 1960) is based on the flow continuity principle, i.e. it assumes that the particles are settling in an environment in which there is an upward velocity and hence their net settling velocity is equal to their unhindered settling velocity, v_t, minus the upward velocity.

If d is the average dimension of the particles and n is the number of particles in a short length Δl, then in any horizontal area Δl^2, there will be $f_1n^2d^2$ area of particles, where f_1 is a shape factor equal to $\pi/4$ for spheres. In any elementary volume Δl^3 there will be $f_2n^3d^3$ volume of particles, where f_2 is a shape factor equal to $\pi/6$ for spheres. As the cloud of particles settles, liquid is displaced at a mean upward velocity v_w, such that

$$v_h = v_t - v_w$$

From continuity:

$$v_h f_1 n^2 d^2 = v_w\left(\Delta l^2 - f_1 n^2 d^2\right)$$

Combining these two relations:

$$v_h = v_t\left(1 - \frac{f_1 n^2 d^2}{\Delta l^2}\right)$$

since

$$c_v = \frac{f_2 n^3 d^3}{\Delta l^3}$$

therefore

$$v_h = v_t\left(1 - \frac{f_1}{f_2^{2/3}} c_v^{2/3}\right)$$

or

$$v_h = v_t\left(1 - K c_v^{2/3}\right) \qquad (2.12)$$

where

$$K = \frac{f_1}{f_2^{2/3}} = 1.21 \text{ for spheres}$$

Bond (1960) found K to have a value of 2.78, on the basis of experiments on alum and lime flocculent suspensions; these suspensions were found to exhibit true hindered settling up to a limiting volumetric concentration of about 0.16. Above this concentration the particles were found to be in partial contact with their neighbours and hence no longer settling as independent entities. The lower c_v limit at which hindering of sedimentation is initiated has been reported to be 0.005 (camp, 1946). As shown above, the gravimetric concentration corresponding to a given volumetric concentration is a function of the specific gravity of the solid matter and the amount of entrained water. Thus, the gravimetric concentration at which hindering of sedimentation is initiated is much lower for porous low-density flocs than it is for impermeable high-density particles.

2.4 SETTLING OF FLOCCULENT SUSPENSIONS

Zone settling is the term used to describe the settling behaviour of highly flocculent suspensions such as activated sludge (generated in biological treatment of wastewaters) or the metal hydroxide precipitates produced in chemical coagulation processes in water treatment. When the concentration of these suspensions exceeds about 1000 mg l^{-1}, the floc particles form a loose interconnected mass which settles as a blanket, forming a distinct interface between the settling mass and the supernatant water.

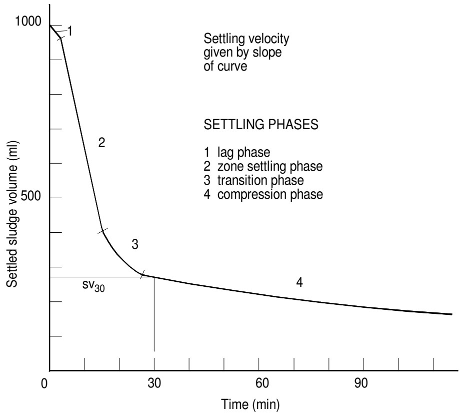

Fig 2.3 **Illustration of zone-settling behaviour in a typical activated sludge settled test in a 1-litre cylinder**

Zone settling behaviour, as typically observed in a batch-settling test of an activated sludge sample, is demonstrated in Fig 2.3, which shows the rate of settling of an activated sludge sample in a 1-litre measuring cylinder as a function of time. Four distinct settling phases are identified. In the initial lag-phase (1), the suspension concentration is effectively uniform throughout the column height. After a short initial lag period, the solids-liquid interface settles at a uniform rate and constant concentration, so-called zone settling (2). After a period the interface settling rate decelerates (transition phase, 3) in response to the increasing interfacial solids concentration. Finally, the settled solids enter a compression phase, in which the particles are in physical contact with each other, thereby slowing down the thickening process. The compression zone begins to develop at the base of the column at the outset of the settling process, gradually expanding upwards and connecting to the zone settling region through an intermediate transition zone.

This type of settling has been analysed by Kynch (1952). His analysis is based on the assumption that the settling velocity v in the zone and transitional settling phases (refer Fig 2.3) is a function only of the gravimetric concentration, that is $v = f(c)$. Consider any thin layer X within the transition zone, having a suspension concentration c, as illustrated in Fig 2.4. Such a layer is bounded on its upper side by a layer of lesser concentration and on its lower side by a layer of higher concentration. The position of this layer moves upward with time. Let its rise rate be u. Since the concentration within layer X remains constant, the influx of solids through the upper boundary must be equal to the efflux of solids through the lower boundary:

$$(c-dc)(v+dv+u) = c(v+u)$$

Neglecting the products of small quantities:

$$u = -v + c\frac{dv}{dc} \tag{2.14}$$

Hence

$$u = -f(c) + cf'(c) = \text{constant}$$

since $v = f(c)$ and c has been assumed as constant. Thus the rate of ascent of any plane of fixed concentration is constant provided the settling of particles within the transition phase is dependent only on concentration.

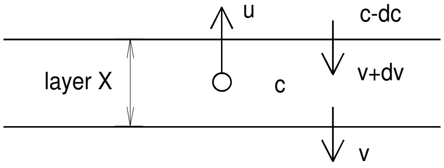

Fig 2.4 **Kynch settling behaviour model**

The relationship between v and c may be obtained from a batch-settling test. The results of a typical batch-settling test on a suspension exhibiting zone-settling behaviour (e.g. activated sludge) are shown in Fig 2.5. At zero time the suspension has a uniform concentration c_0 and an initial height h_0. In a settling time t_2, the solids–water interface has dropped to a height h_2; let the concentration at time t_2 be c_2. If A is the column cross-sectional area, the total mass of solids in the column, c_0h_0A, will have passed through the plane of concentration c_2 as it ascended from the base of the vessel with a velocity u_2. Hence:

$$c_0h_0A = c_2(v_2 + u_2)At_2 \tag{2.15}$$

The velocity of ascent, u_2, has already been shown to be constant and is therefore equal to h_2/t_2; hence

$$c_0h_0 = c_2t_2\left(v_2 + \frac{h_2}{t_2}\right) \tag{2.16}$$

The settling velocity, v_2, is given by the slope of the curve at the point (h_2, t_2) and from the diagram is seen to be equal to $(h_1 - h_2)/t_2$; hence

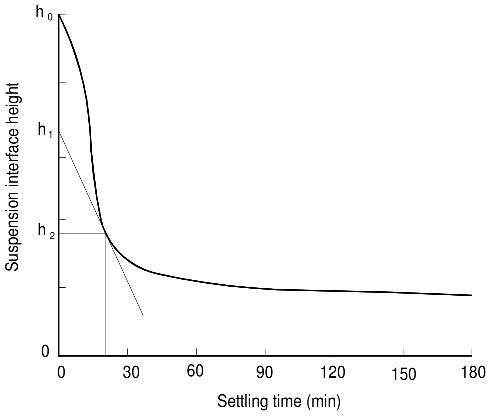

Fig 2.5 **Batch-settling test on an activated sludge sample**

$$c_0 h_0 = c_2 t_2 \left[\left(\frac{h_1 - h_2}{t_2} \right) + \frac{h_2}{t_2} \right] \tag{2.17}$$

Therefore

$$c_0 h_0 = c_2 h_1$$

and

$$c_2 = c_0 \frac{h_0}{h_1} \tag{2.18}$$

Thus from a batch-settling test the settling velocity corresponding to any given concentration can be computed. It is also clear from the above that h_1 is the height the sludge would occupy if it were uniformly distributed at a concentration c_2.

2.5 SETTLING VELOCITY DISTRIBUTION

The distribution of settling velocity in a discrete particle suspension is determined experimentally in a laboratory settling column test. Laboratory settling columns are usually clear plastic tubes having a diameter of at least 100mm and a length of about 2m, with a set of sampling points distributed over the column height. Care is taken to ensure that the suspension is uniformly distributed at the start of the test.

Table 2.1 contains a set of concentration measurements in samples taken during a laboratory column settling test from a sampling point located 1m below the water surface. From this set of results it can be seen that 95% of particles have a settling velocity of less than 1.0 m in 30 min or 2 mh^{-1} and that 83% have a settling velocity of less than 1 mh-1, etc. The cumulative distribution of settling velocities for this suspension is plotted in Fig 2.6. For truly discrete settling, the depth h of the sampling point does not affect the resultant settling velocity distribution curve. If the suspension is flocculent, however, a different velocity distribution will be found for each sampling point.

In many cases, even in relatively dilute suspensions, particles coalesce to form particle aggregates having increased settling velocities. The extent of such flocculation is a function of many variables including suspension type and concentration, the prevailing velocity gradients and time. The expected removal of a

Table 2.1

Time (min)	0	30	60	90	120	180	360
Concentration (mg l-1)	42	40	35	29	22	10	2.5
% of initial concentration	100	95	83	69	52	24	6

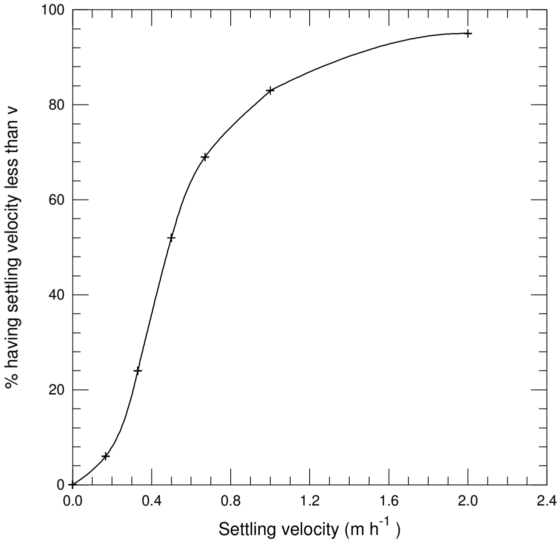

Fig 2.6 **Settling velocity distribution for discrete particles**

Flocculent suspension in a sedimentation process can be estimated from a laboratory settling test using a suspension column height equal to that used in the process. At various time intervals, samples are withdrawn from ports at different depths and analysed for suspended solids. The percentage removal is computed for each sample and is plotted in a graph against time and depth. Curves of equal percentage removal are then interpolated between the plotted points, as illustrated in Fig 2.7. The resulting curves can be used to determine the overall removal of solids for any detention time and depth within the range of the data, bearing in mind that the test conditions are quiescent. For example, for a detention time of t_2 and a

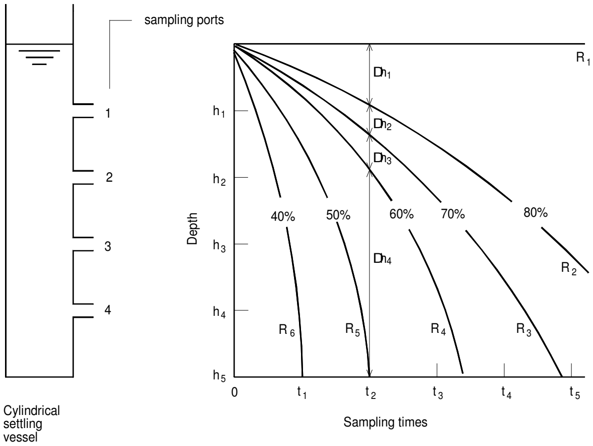

Fig 2.7 **Typical settling curves fir flocculent suspensions**

depth of h_5 the removal is equal to

$$\frac{\Delta h_1}{h_5} x \left(\frac{R_1+R_2}{2}\right) + \frac{\Delta h_2}{h_5} x \left(\frac{R_2+R_3}{2}\right) + \frac{\Delta h_3}{h_5} x \left(\frac{R_3+R_4}{2}\right) + \frac{\Delta h_4}{h_5} x \left(\frac{R_4+R_5}{2}\right) \tag{2.19}$$

2.6 NATURE OF SUSPENSIONS IN WASTEWATER TREATMENT

The suspensions encountered in wastewater treatment include the raw wastewater itself, the humus and activated sludge suspensions produced in biological treatment processes, and the flocculent suspensions generated by physicochemical treatment.

2.6.1 Raw wastewater

Raw municipal wastewater, which is a mixture of domestic wastewater (sewage) and industrial wastewater, varies in composition from hour to hour, day to day, and location to location. The total solids (TS) content of raw wastewater (i.e. the sum of the dissolved, colloidal and suspended species of solid matter) depends on the per capita water consumption and the nature of the industrial discharges in the sewer catchment area. The TS value typically varies within the range 500-1000 mg l^{-1}. A typical fractional characterisation of the solid matter in raw municipal wastewater as organic, inorganic, dissolved and suspended species is presented in Fig 2.8.

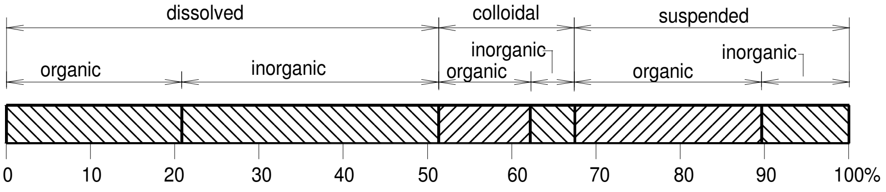

Fig 2.8 Typical proportional distribution of solid matter in municipal sewage

The inorganic particulate fraction of municipal wastewater, commonly referred to as *grit*, which typically has a much higher specific gravity and settling velocity than the organic particulate fraction, is conventionally selectively separated in the first stage of treatment. The remaining particulate matter varies in size from 10 mm to colloidal size. Excluding the grit fraction, the specific gravity of the dry solids is in the range 1.0 – 1.5 but wet solids contain a high proportion of water bringing their specific gravities close to 1.0. Roughly 55-65% of the total suspended solids (TSS) in municipal wastewaters are settleable and hence removable by a gravity sedimentation process commonly called primary sedimentation. The nature and volumetric concentration of settleable solids in sewage are such that their settling in primary sedimentation processes may be categorized as hindered flocculent settling. Primary sedimentation generates a solids residue or primary sludge which can be readily thickened by gravity to achieve a total dry solids concentration in the range 4-8% by weight.

The TSS fraction of raw municipal wastewater typically amounts to about 72g per person equivalent (PE).

2.6.2 Activated sludge

Activated sludge (AS) is the microbial suspension or so-called ‘mixed liquor’ generated in aerobic suspended floc biological wastewater treatment processes. It is more consistent in quality and concentration than raw wastewater but varies somewhat with process operating conditions. The AS particles or flocs range in size from a few microns to more than 1 mm. The specific gravity of the dry solids is over 1.5 but, since this is an organic flocculent suspension containing much entrained water, the insitu specific gravity ranges from 1.01 to 1.03. The operating mixed liquor concentration in AS processes may vary in the range 1500-5000 mg l^{-1}; being typically highly flocculent, it exhibits zone-settling behavior as illustrated in Fig 2.4. However, some activated sludges flocculate poorly and hence have very low settling velocities. This condition is termed ‘bulking’, a condition that adversely affects its separation by sedimentation processes, as discussed in Chapter 12.

2.6.3 Humus sludge

Humus sludge is the microbial biofilm residue generated in aerobic attached film biological wastewater treatment processes (aerobic biofiltration). The suspension varies in particle size from colloidal to macroscopic with considerable fluctuation in nature and concentration which are subject to seasonal change. The typical concentration range is 50-150 mg l^{-1} with a specific gravity similar to the organic fraction of raw wastewater. The percentage of suspended solids that are settleable is usually in the region of 70-80%.

Most wastewater suspensions include colloidal particles which are not amenable to separation by the conventional sedimentation and sand filtration processes. The characteristics of colloidal suspensions and the processes used for their removal are discussed in Chapter 3.

2.7 NATURE OF SUSPENSIONS IN WATER TREATMENT

The colloidal impurities in natural surface waters are made up of a complex mixture of particles derived mainly from non-consolidated catchment deposits such as soil, clays and peat. Depending on the degree of contamination, surface waters may also contain colloids from domestic and industrial wastes, live and decaying algae and their decomposition products, bacterial cells, decaying organic matter, and colour colloids. Some of the colloidal impurities, such as clay, may be hydrophobic in nature, whereas others, like some sewage colloids, may be hydrophilic.

As shown in Fig 1.2, colloidal particles range in size from 1 to 10^{-3} μm. Clay-derived colloidal particles tend to be at the upper end of this size range, whereas colour colloids are more likely to be at the lower end (Black and Christman, 1963).

Because of their small size, these colloidal particles are not visible under an ordinary microscope, even of the highest power. They may be viewed by electron microscopy or by means of the Tyndall effect. Their concentration is indirectly quantified by turbidity measurement (Standard Methods, 2005) or by Secchi disc transparency reading, both of which measure the extent to which colloidal particles interfere with the transmission of light.

The process of coagulation leads to the destabilization and aggregation of colloidal particles into hydroxide floc particles of size up to 1 mm and above, of increased density and settling velocity. The floc particles are of loose structure and contain much entrained water. The coagulating compounds have the following dry densities:

Al oxide	$Al_2O_3(20\ H_2O)$	1180 kg m^{-3}
Fe oxide	$Fe_2O_3(20\ H_2O)$	1340 kg m^{-3}
Crystalline	$CaCO_3$	2600 kg m^{-3}

Their wet densities depend on the amount of entrained water and may be calculated using equation (1.16).

The sludge mass produced by chemical coagulation processes is a function of the cation dose and the amount of colloidal matter removed; it may be approximately estimated (Hall & Hyde, 1992) as follows:

Sludge solids (mg l^{-1} treated water) = 2 x turbidity removed (NTU)
+ 0.2 x colour removed (^{o}H)
+ 2.9 x aluminium precipitated (mg l^{-1} Al)
+ 1.9 x iron precipitated (mg Fe l^{-1})

The corresponding floc volume may vary in the range 100-300 ml floc g^{-1} cation (Hudson,1965).

In experiments in upflow sludge blanket clarification, Ives and Hale (1970) found that the solids concentration in the sludge blanket was inversely related to the upflow rate. The volumetric concentration, measured by noting the sludge volume after 3 h quiescent settling, was found to vary from 0.2 to 0.05, corresponding to nominal upflow rates ranging from 1 to 2.5 m h^{-1}.

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